B.

$t=0$, $y=2$ given.

$t=4$

$$y = -\frac{5}{0.7^2}\sin(0.7(4)) + \frac{5(4)}{0.7} - 5(4) + 2$$

$$= 7.153 \text{ ft.}$$

$$\delta y = y(4) - y(0) = 5.153 \text{ ft}$$

c.

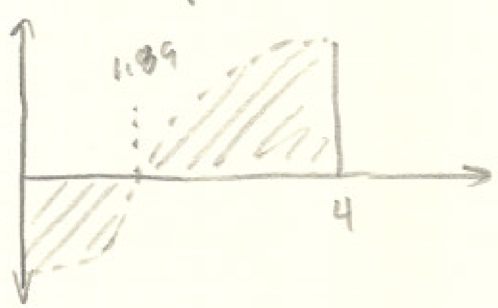
$$s = \int_0^4 |v(t)|\,dt$$

$$v = -5 + \frac{5}{0.7^2} - \frac{5}{0.7}\cos 0.7t$$

$$v = 2.143 - 7.143\cos 0.7t$$

let $v=0$ then $t = 1.809$

draw a picture.



$$\therefore \quad s = -\int_0^{1.809} v(t)\,dt + \int_{1.809}^4 v(t)\,dt$$

$$= -y\Big|_0^{1.809} + y\Big|_{1.809}^4$$

$$= -\left(y(1.809) - y(0)\right) + \left(y(4) - y(1.809)\right)$$

$s = 16.87$ ft.

EX. A particle has the acceleration $a(x) = -4x \, m/s^2$
The velocity of the particle is 2 m/s when it passes through the origin

A. $v(x) = ?$

B. if the particle is in the origin when $t=1$, determine $x(t)$, $v(t)$, $a(t)$

Solution.

A.

$$a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$$

$$a\,dx = v\,dv$$

$$\frac{1}{2}v^2 = \int -4x \; dx$$

$$\frac{1}{2}v^2 = -2x^2 + C$$

$$x = 0 \quad , \quad v = 2$$

$$\frac{1}{2}(2)^2 = -2(0)^2 + C$$

$$\therefore C = 2$$

$$\frac{1}{2}v^2 = -2x^2 + 2 = 2(1 - x^2)$$

$$v = 2\sqrt{1 - x^2} \qquad |x| \le 1$$

B.

$$v = 2\sqrt{1-x^2}$$

$$\frac{dx}{dt} = 2\sqrt{1-x^2}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \int 2\,dt$$

$$\sin^{-1} x = 2t + C$$

$$x = \sin(2t + C)$$

$t = 1 \quad x = 0$

$$0 = \sin(2(1) + C)$$

$$\therefore C = -2$$

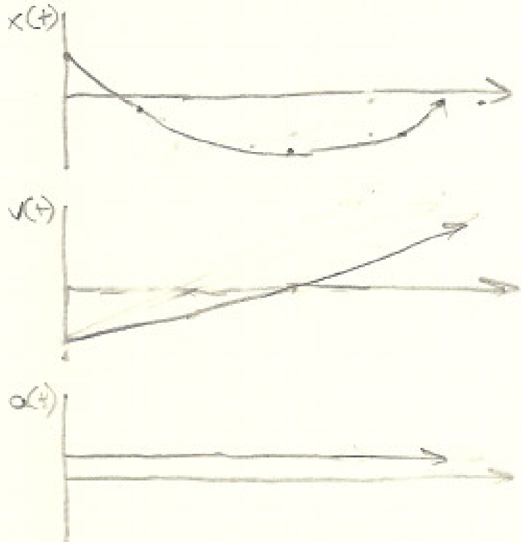
$$x = \sin(2(t+1))$$

EX.

Given $x(t) = 5t^2 - 30t + 20$ m.

Determine the velocity and acceleration as a function of time. Determine the distance travelled by the particle between $t=0$ and $t=5s$

$v(t) = 10t - 30$ m/s



$a(t) = 10$ m/s²

$$d = \int_0^5 |v(t)|\,dt$$

$$= -\int_0^3 (10t-30)\,dt + \int_3^5 (10t-30)\,dt.$$

$$= (-5t^2 + 30t)\Big|_0^3 + (5t^2 - 30t)\Big|_3^5$$

$$= -\left[5(3)^2 - 30(3) + 5(0) - 30(0)\right] + \left[5(5)^2 - 30(5) - 5(3)^2 + 30(3)\right]$$

$$= 95 \text{ m}$$

EX.

Given $V(t) = -4t^2 + 40t - 70$ (m/s) $\quad x(0) = 20$ m

Determine $x(t)$, $a(t)$

Determine the distance traveled between $t=5$, $t=8$

$$x(t) = \int V(t)\,dt = -\frac{4}{3}t^3 + 20t^2 - 70t + 20 \text{ (m)}$$

$a(t) = -8t + 40 \; (m/s^2)$

$v(t) = 0 = -4t^2 + 40t - 70$

roots $= 2.621\,s$ & 7.739

$$s = \int_5^8 |v(t)|\,dt$$

$$= -\int_5^{7.739} v(t)\,dt + \int_{7.739}^{8} v(t)\,dt.$$

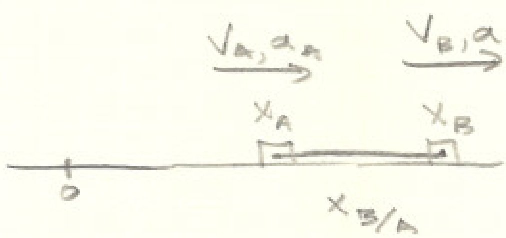
$= 55.5\,m.$

§13.4 REALITIVE MOTION ALONG A LINE.

n particles, m cooridantes to identify the motion ($m \leq n$) the system has m degrees of freedom.

if $m = n$ independant realitive motion

$m < n$ dependant realitive motion.

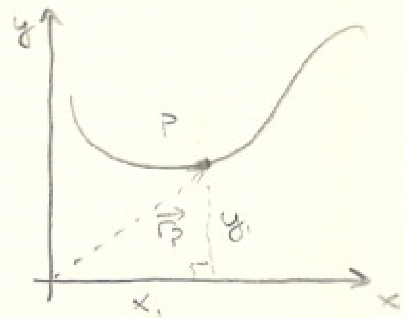


$x_B = x_A + x_{B/A}$ $\qquad$ $x_{B/A} = x_B - x_A$

$v_B = v_A + v_{B/A}$ $\qquad$ $v_{B/A} = v_B - v_A$

$a_B = a_A + a_{B/A}$ $\qquad$ $a_{B/A} = a_B - a_A$

§ 13.5 PLANE LINEAR MOTION.



$$\vec{r}_P = x\vec{i} + y\vec{j}$$

$$\vec{v}_P = \frac{d\vec{r}_P}{dt} = \dot{x}\vec{i} + \dot{y}\vec{j}$$

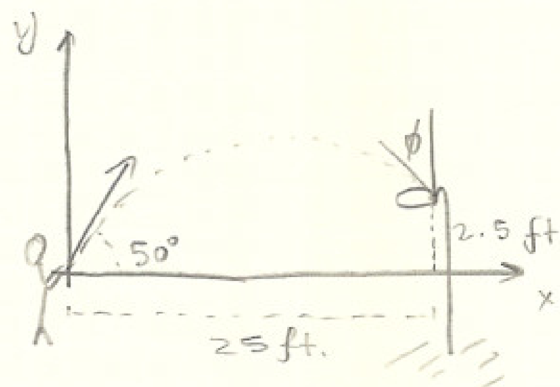
$$\vec{a}_P = \frac{d\vec{v}_P}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$$

EX. A Basket ball player shoots the wall at the basket which is 25 ft away. The acceleration of the ball during flight is 32.2 ft/s downward

Determine:

A. The required initial velocity?

B. The angle ϕ that the trajectory of the ball will make with the vertical when the ball goes through the basket.



Solution:

$$a(t) = -32.2 ft/s^2 \vec{j}$$

$$a(t) = \ddot{x}\vec{i} + \ddot{y}\vec{j}$$

$$\therefore \quad \ddot{x} = 0 \qquad \ddot{y} = -32.2\vec{j}$$

$$\dot{x} = c_1 \qquad \dot{y} = -32.2t + d_1$$

$$x = c_1 t + c_2 \qquad y = -16.1t^2 + d_1 t + d_2$$

when we set $t=0$

$$\dot{x} = V_0 \cos 50^\circ \qquad \dot{y} = V_0 \sin 50^\circ$$

when t is not set

$$\dot{x} = V_0 \cos 50^\circ \qquad \dot{y} = -32.2 + V_0 \sin 50^\circ$$

when the ball goes through the basket

$$x = V_0 t \cos 50 = 25$$

$$y = -16.1t^2 + V_0 t \sin 50^\circ = 7.5$$

now we can solve.

$$t = 1.302 \text{ s}$$

$$V_0 = 29.87 \text{ m/s}.$$

when we find the velocity at 1.302 s and break the velocity into x and y components we can find ϕ

$V_x = 29.87 \cos 50^\circ$

$V_y = -32.2(1.309) + 29.87 \sin 50^\circ$

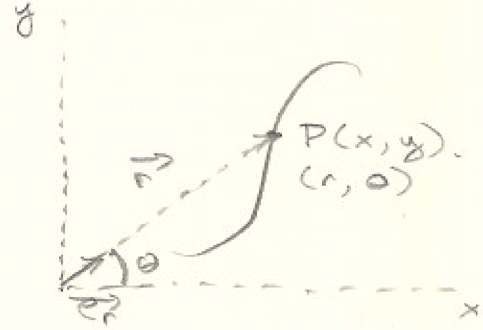
$\tan \phi = \frac{V_x}{V_y}$

$\phi = 45.24^\circ$

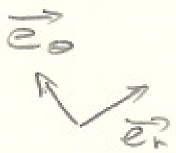
A#5 (DUE JULY 24)

CH 9
1, 2, 7, 13
CH 13
2, 7, 16, 88, 91, 104, 120

POLAR COORDINATE (RADIAL / TRANSVERSE COORDINATES)



if we rotate $\vec{e_r}$ 90° we can define it as $\vec{e_\theta}$



$$\dot{\vec{i}} = \dot{\vec{j}} = 0$$

$$\dot{\vec{e_r}} \neq \dot{\vec{e_\theta}} \neq 0$$

$$\vec{r_P} = r_P \vec{e_r}$$

$$\vec{e_r} = \cos\theta \vec{i} + \sin\theta \vec{j}$$

$$\vec{e_\theta} = -\sin\theta \vec{i} + \cos\theta \vec{j}$$ (90° degrees from $\vec{e_r}$)

$$\dot{\vec{e_r}} = -\dot\theta \sin\theta \vec{i} + \dot\theta \cos\theta \vec{j} = \dot\theta \vec{e_\theta}$$

$$\dot{\vec{e_\theta}} = -\dot\theta \cos\theta \vec{i} - \theta \sin\theta \vec{j} = -\dot\theta \vec{e_\theta}$$

VELOCITY

$$\vec{V_P} = \dot{\vec{r_P}} = \dot r \vec{e_P} + r \dot{\vec{e_P}}$$

$$= \dot r \vec{e_r} + r \theta \vec{e_\theta}$$

$V_r = \dot r_P \quad V_\theta = r\dot\theta$

$$\vec{A_p} = \dot{\vec{V_p}} = \ddot{r}\vec{e_r} + \dot{r}\dot{\vec{e_r}} + (\dot{r\dot{\theta}})\vec{e_\theta} + r\dot{\theta}\dot{\vec{e_\theta}}$$

$\theta\vec{e_r}$ $\qquad$ $-\theta\vec{e_r}$

$\dot{r}\dot{\theta} + r\ddot{\theta}$

Special case of a circle.

$r = \text{constant}$

$\dot{r} = \ddot{r} = 0$

$V_r = 0 \quad V_\theta = r\dot{\theta}$

$A_R = -r\dot{\theta}^2 \quad A_\theta = r\ddot{\theta}$

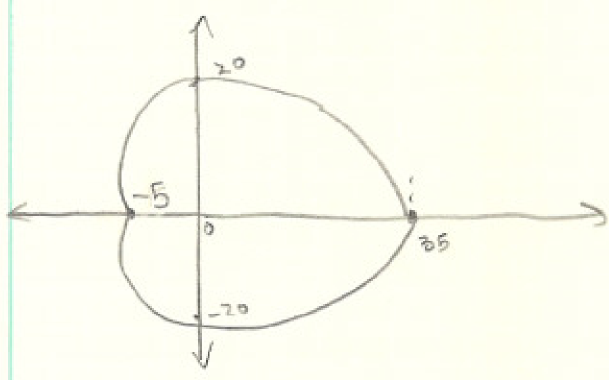
$$\left.\begin{aligned} A_R &= \ddot{r} - r\dot{\theta}^2 \\ A_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{aligned}\right\}$$

Ex. A particle is following a path given by

$r(\theta) = 20 + 15\cos\theta$

Given that $\dot{\theta} = 30$ rev/min and that $\theta = 0$ when $t = 0$

Determine the velocity and acceleration of the particle at $t = 0.75$ s



Solution:

$$\dot{\theta} = 30 \text{ rev/min} \cdot \frac{60 \text{ sec/min}}{2\pi \text{ rad/rev}} = \pi \text{ rad/s}$$

$$\ddot{\theta} = 0$$

$$\theta = \int \pi \, dt$$

$= \pi t + c$ since there is no initial condition

$= \pi t$

$r = 20 + 15\cos(\pi t)$ subing in πt for θ

$$\dot{r} = -15\pi \sin(\pi t)$$

$$\ddot{r} = -15\pi^2 \cos(\pi t)$$

$$V_r = \dot{r} = -15\pi \sin(\pi t)$$

$$V_\theta = r\dot{\theta} = \pi(20 + 15\cos(\pi t)$$

$$A_r = \ddot{r} - r\dot{\theta}^2 = -15\pi^2\cos(\pi t) - \pi^2(20 + 15\cos(\pi t))$$

$$= -\pi^2(20 + 15\cos(\pi t))$$

$$A_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -30\pi^2 \sin(\pi t)$$

when $t = 0.75$

$V_R = -33.3$ mm/s

$V_\theta = 29.5$ mm/s

$A_r = 11.97$ mm/s

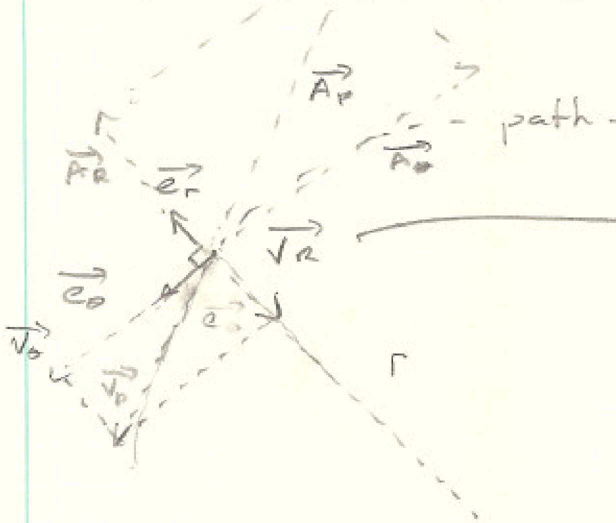
$A_\theta = -209.4$ mm/s.

$\theta = 135°$

$$\vec{r_P} = r\,\vec{e_r}$$

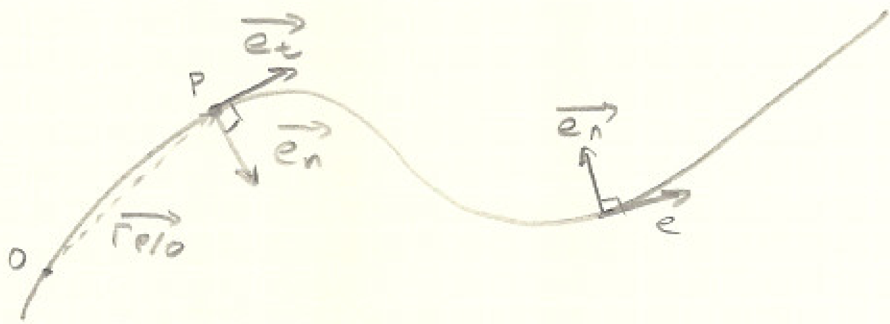
$$\vec{V_P} = V_R\,\vec{e_r} + V_\theta\,\vec{e_\theta}$$

$$\vec{A_P} = A_R\,\vec{e_R} + A_\theta\,\vec{e_\theta}$$



going opposite way b/c neg.

NORMAL AND TANGENTICAL COORDINATES



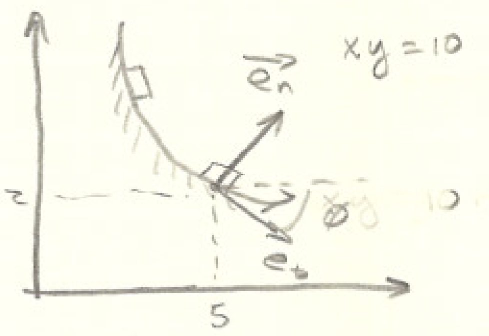
$\vec{e_n}$ points to the center of curvature, $\vec{e_t}$ points in the direction of the tangent, and direction of motion.

$$\vec{V_P} = V\,\vec{e_t} \qquad V = \dot{s} = \frac{ds}{dt}$$

$$\vec{A_P} = \dot{\vec{V_P}} = \dot{V}\,\vec{e_t} + \frac{V^2}{\rho}\,\vec{e_n}$$

ρ: is the radious of the curvature.

EX.



When the box reaches the point $x = 5m$, it has a speed of $5 m/s$ and speed is d/c at a rate of $0.5 \; m/s^2$.

Determine the tangental and normal components of the accelleration of the box.

Solution:

$$\vec{A_P} = \dot{v}\,\vec{e_t} + \frac{v^2}{\rho}\,\vec{e_n}$$

$$A_t = \dot{v} \qquad A_N = \frac{V^2}{\rho}.$$

At $x = 5m \quad V = 5 m/s \quad \dot{v} = -0.5 \; m/s^2$

to find radius of the curve

$$\frac{1}{\rho} = \frac{|y''|}{(1 + (y')^2)^{3/2}}$$

$$y = \frac{10}{x} \qquad y' = -\frac{10}{x^2} \qquad y'' = \frac{20}{x^3}$$

at $x = 5 \quad y = 2 \quad y' = -0.4 \quad y'' = 0.16$

$$\frac{1}{\rho} = \frac{0.16}{(1 - (-0.4)^2)^{3/2}} = 0.1281 \; (m^{-1})$$

$$A_n = \frac{V^2}{\rho} = 25(0.1281 \; m^{-1}) = 3.20 \; m/s^2$$

$$\tan(180 - \phi) = \left.\frac{dy}{dx}\right|_5 = -0.4$$

$\therefore \; \phi = 21.80°$

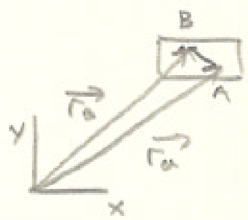
$\vec{A_t} = 0.5 \; m/s^2 \quad 21.8°$

$\vec{A_n} = 3.20 \; m/s^2 \quad 68.2°$

CHAPTER 14 KINEMATICS OF RIGID BODY.

translation:

the orientation of every straight line is fixed.

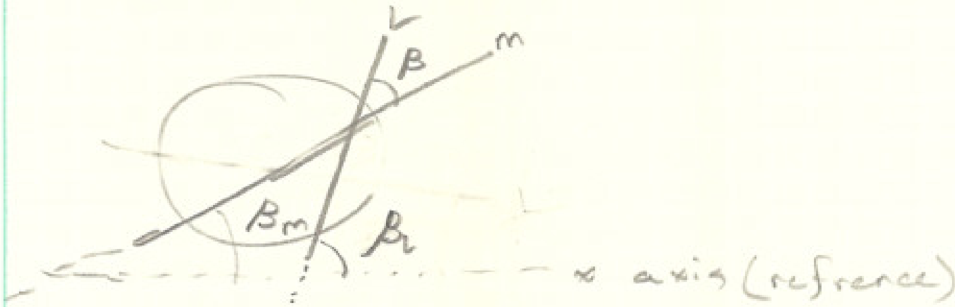


$\vec{r}_{B/A}$: if fixed when the body is in motion.

$$\dot{\vec{r}}_{B/A} = \ddot{\vec{r}}_{B/A} = 0$$

$$\therefore \quad \dot{\vec{r}}_B = \dot{\vec{r}}_A \Rightarrow \vec{V}_B = \vec{V}_A$$
$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A \Rightarrow \vec{A}_B = \vec{A}_A$$

PLANAR MOTION



$\beta_m = \beta_m(t)$

$\beta_L = \beta_L(t)$

β is a constant.

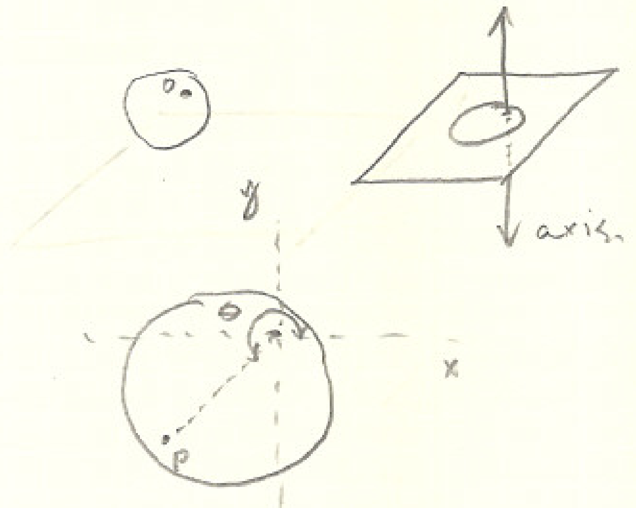
$\beta_m = \beta_L + \beta$.

$\dot{\beta}_m = \dot{\beta}_L$

$\omega = \dot{\beta}_m = \dot{\beta}_L$ this is the angular velocity of the body

$\alpha = \ddot{\beta}_m = \ddot{\beta}_L$ this is the angular acceleration of the body

ROTATION ABOUT A FIXED AXIS.



$\omega = \dot{\theta}$

$\alpha = \ddot{\theta}$

the path of point P. will be a circle about O. therefore we with use radial coordinates.

$\vec{r_P} = r\vec{e_r}$

$\vec{V_P} = \dot{\vec{r_P}} = r\dot{\theta}\vec{e_\theta} = r\omega\vec{e_\theta}$

$\vec{\omega} = \omega\vec{k}$

$\vec{V_P} = r\omega(\vec{k} \times \vec{e_r}) = (\omega\vec{k}) \times r\vec{e_r} = \vec{\omega} \times \vec{r_P}$

$\vec{A_P} = \dot{\vec{V_P}} = \dot{\vec{\omega}} \times \vec{r_P} + \vec{\omega} * \dot{\vec{r_P}}$

$= \vec{\alpha} \times \vec{r_P} + \vec{\omega} \times (\vec{\omega} \times \vec{r_P})$

we know

$\vec{\alpha} = \alpha\vec{k}$

$\vec{r_P} = r\vec{e_R}$

$$\therefore \vec{\alpha} \times \vec{r_p} = \alpha \vec{k} \times r\vec{e_R} = r\alpha\vec{e_\theta}$$

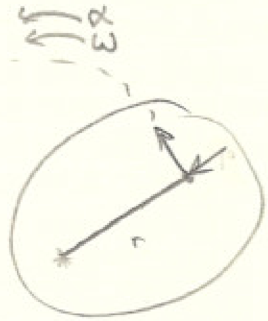
$$\omega\vec{k} \times (\omega\vec{k} \times r\vec{e_P}) = r\omega^2 \vec{k} \times \vec{e_\theta}$$
$$= r\omega^2(-\vec{e_r})$$

$$\therefore \vec{A_P} = r\alpha\vec{e_\theta} - r\omega^2\vec{e_r}$$
$$= r\alpha\vec{e_t} + r\omega^2\vec{e_n}$$

$$\vec{e_\theta} = \vec{e_t}$$
$$\vec{e_r} = -\vec{e_n}$$

$$= A_t\vec{e_t} + A_n\vec{e_n}$$

$$\therefore A_t = r\alpha$$
$$A_n = r\omega^2$$

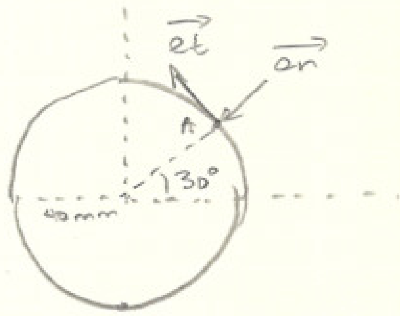


$$\vec{V_p} = V_p \vec{e_t} \qquad V_p = \omega r$$

$$\vec{A_p} = \vec{A_{pt}} + \vec{A_{pN}}$$
$$= A_{pt}\vec{e_t} + A_{pn}\vec{e_n}$$

$$A_{pt} = r\alpha \qquad A_{pN} = r\omega^2$$

Ex. Determine the acceleration of a pt A at the instant



$$\omega = 2 \text{ rad/s}$$
$$\alpha = 1 \text{ rad/s}^2$$

Solution:

$$\vec{e_n} = -\cos 30° \vec{i} - \sin 30° \vec{j}$$
$$\vec{e_t} = -\cos 60° \vec{i} + \sin 60° \vec{j}$$

$$r = 0.04 \text{ m}$$

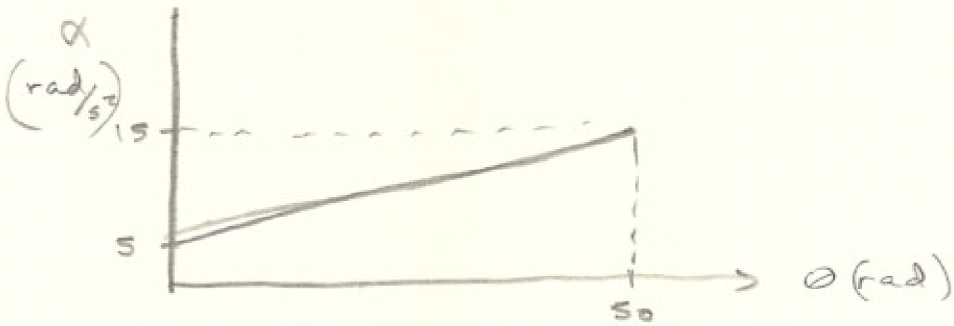
$$A_t = r\alpha = 0.04(1) = 0.04 \text{ m/s}^2$$

$$A_N = r\omega^2 = 0.04(2)^2 = 0.16 \text{ m/s}^2$$

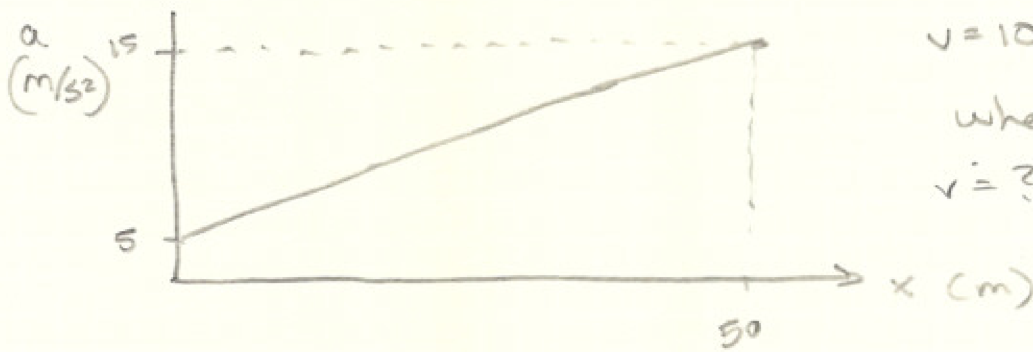
$$\vec{A_A} = 0.04(-\cos 60° \vec{i} + \sin 60° \vec{j}) + 0.16(-\cos 30° \vec{i} - \sin 30° \vec{j}$$
$$= -0.1586\vec{i} - 0.04536\vec{j}$$

$$A_A = 0.1649 \text{ m/s}^2 \quad 15.96°$$

EX. A disk is rotating at a rate of 10 rad/s when $\theta = 0$, the angular acceleration of the disk varies linearly with the angular position as shown. Determine the angular velocity of the disk when $\theta = 50$ rad.



let us look at another problem.



$v = 10$ m/s

when $x = 0$

$v = ?$ at 50 m

we can find the line

$$a(x) = \frac{1}{5}x + 5$$

$$a(x) = \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt} = V\frac{dV}{dx}$$

$$\int a(x)\,dx = \int V\,dV = \frac{1}{2}V^2$$

$$V^2 = 2\int \left(5 + \frac{x}{5}\right) dx = 10x + \frac{x^2}{5} + 10^2$$

now we can go back and look at the other problem

$$\alpha(\theta) = 5 + \frac{\theta}{5}$$

$$\alpha(\theta) = \frac{d\omega}{d\theta} \qquad \omega = \frac{d\theta}{dt}$$

$$\alpha(\theta) = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d\omega}{d\theta}$$

$$\int \alpha(\theta)\, d\theta = \int \omega\, d\omega = \frac{1}{2}\omega^2$$

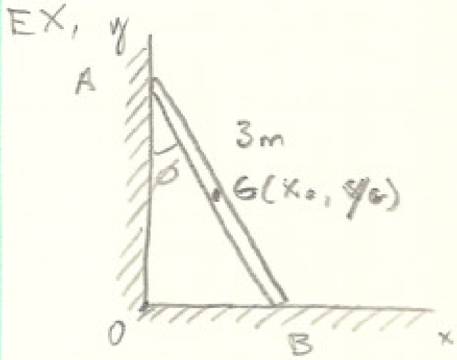
$$\omega^2 = 2\int \left(5 + \frac{\theta}{5}\right) d\theta = 10\theta + \frac{\theta^2}{5} + 100$$

$\therefore$ when θ is 50, ω is 33.17 rad/s.

GENERAL PLANE MOTION.

It is a superposition of translation and fixed axis

EX,



The 3m long ladder AB slides along a corner. At some instant of time, the lower end of the ladder is 1.2 m from the corner and it is moving to the right at a constant speed of 0.5 m/s For the instant, determine

A. the angular accelleration & velocity of the ladder

B. The acceleration $\overrightarrow{A_G}$ of the mass center of the ladder.

SOLUTION:

$$x_B = 3\sin\phi$$

$$y_A = 3\cos\phi$$

$$V_B = \frac{dx_B}{dt} = 3\dot{\theta}\cos\theta = 3\omega\cos\theta$$

$$A_B = \dot{V}_B = 3\alpha\cos\theta - 3\omega^2\sin\theta$$

When $x_B = 1.2\,m$ $V_B = 0.5\,m/s$ $A_B = 0$

$$\sin\theta = \frac{1.2}{3} = 0.4 \quad \theta = 23.58^\circ$$

$$3\omega\cos\theta = 0.5$$

$$3\alpha\cos\theta - 3\omega^2\sin\theta = 0$$

$$\omega = \frac{0.5}{3\cos\theta} = 0.189 \text{ rad/s}$$

$$\alpha = 0.01443 \text{ rad/s}^2$$

B. $x_G = 1.5\sin\theta$

$y_G = 1.5\cos\theta$

$V_{Gx} = \dot{x}_G = 1.5\,\omega\cos\theta$

$A_{Gx} = \dot{V}_{Gx} = 1.5\alpha\cos\theta - 1.5\omega^2\sin\theta = 0$

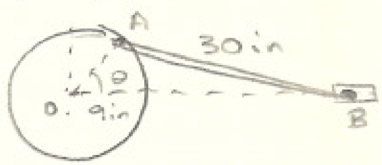
$V_{Gy} = \dot{y}_G = -1.5\omega\sin\theta$

$A_{Gx} = \dot{V}_{Gy} = -1.5\alpha\sin\theta - 1.5\omega^2\cos\theta$

$= -0.05412\ m/s^2$

$\overrightarrow{A_G} = -0.05412\ m/s^2 \downarrow$

EX. 14-9.



Determine the velocity of slider B and the angular velocity of the crank arm AB when $\theta = 60^\circ$

$\omega = 10$ rad/s

$\alpha = 0$